

Probability of Compound Events

- What is the complement of an event?
- What are disjoint events?
- How do we determine the probability of the union of two events or the intersection of two events?

Complement of an Event

- Given an event E , the complement of event E can be denoted as
 - E^c
 - E'
 - $\bar{E}$
- NOTE: In this PowerPoint, we will use E' to denote the complement of the event E .

Complement of an Event

- Given an event E , the complement of event E , E' , is the event that consists of all simple events in the sample space that are not simple events in event E .

Complement of an Event

- That is, given an event E , the complement of event E , E' , is the event containing all simple events that are not contained in event E .

Complement of an Event

- You may find it helpful to think of event E' as the set of simple events *not in E or outside of E* .

Complement of an Event

- We can use a Venn Diagram to picture this.

What is a Venn Diagram?

What is a Venn Diagram?

- A Venn Diagram is a picture representation that is used to represent sets.
- For probability,
 - We use a rectangle to represent the sample space, S .
 - We use circles to represent events such as E .

Venn Diagram for the Sample Space

- The sample space S .



What is a Venn Diagram?

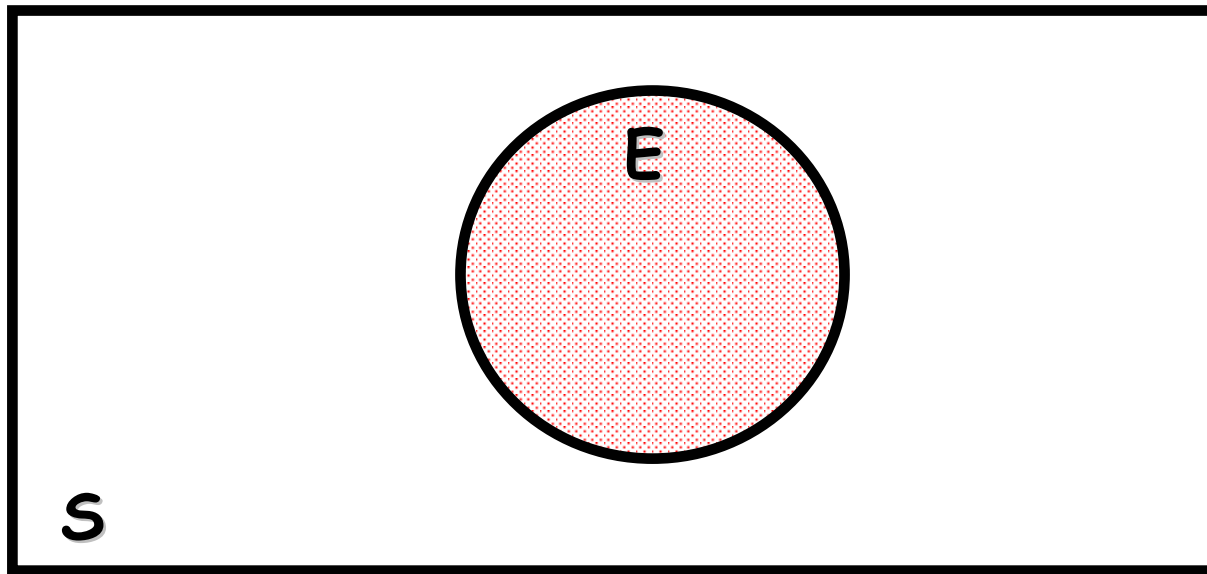
- We use a circle and the region inside the circle placed within the rectangle to represent an event.

What is a Venn Diagram?

- To represent the event E in the sample space S ,
 - We create a circle inside the rectangle to define the space for the event E
 - We shade the inside to the circle to represent the event E
 - We label the circle as E

Venn Diagram for an Event

- The event E .

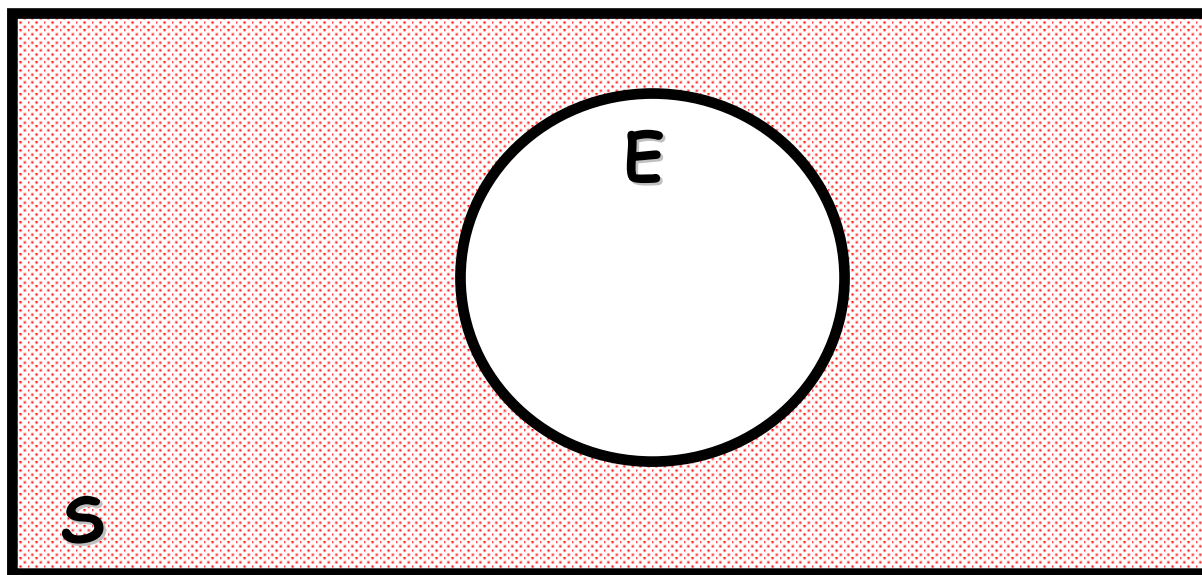


Complement of an Event

- To create the Venn Diagram for the complement of event E , E' , we shade the region outside the circle representing the event E .

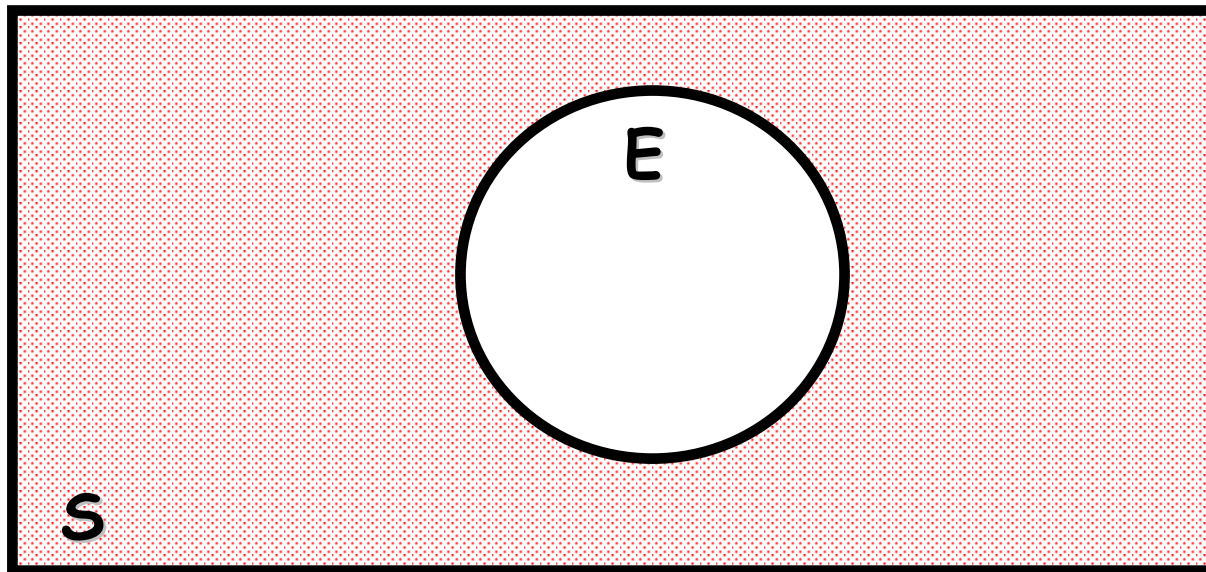
Venn Diagram for the Complement of an Event

- The complement of event E , E' .



Venn Diagram for the Complement of an Event

- The complement of event E , E' .
NOTE: We shade the region that is *not in E* or the region *outside E* .



Example

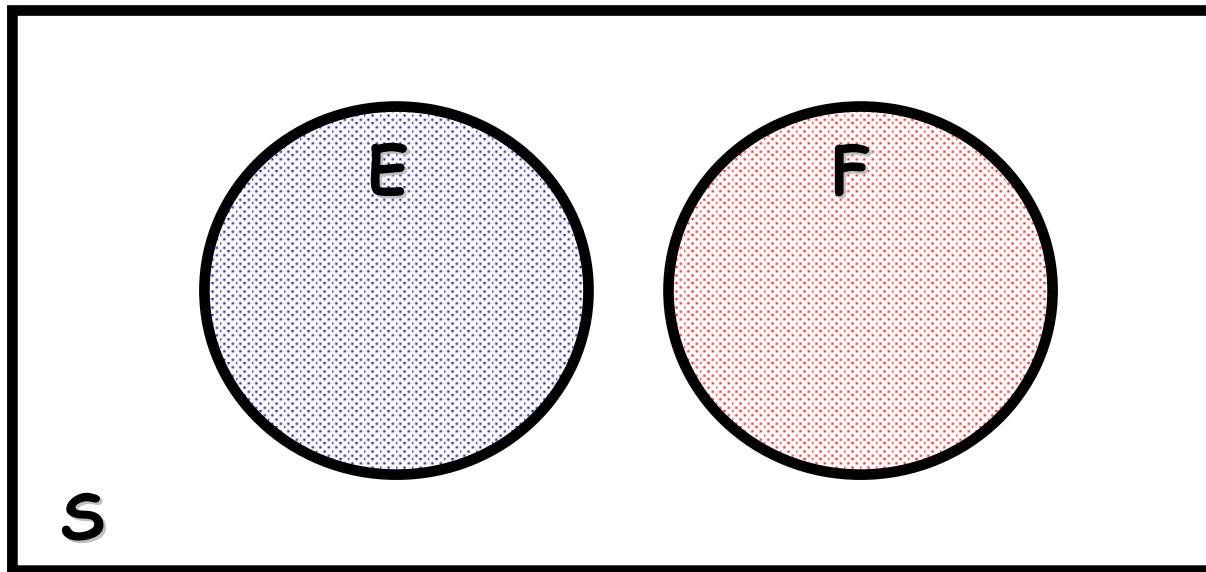
- A single die is rolled. What is the probability that the die does not display a five?

Disjoint Events

- Two events, E and F , are disjoint or mutually exclusive if they have no simple events in common.

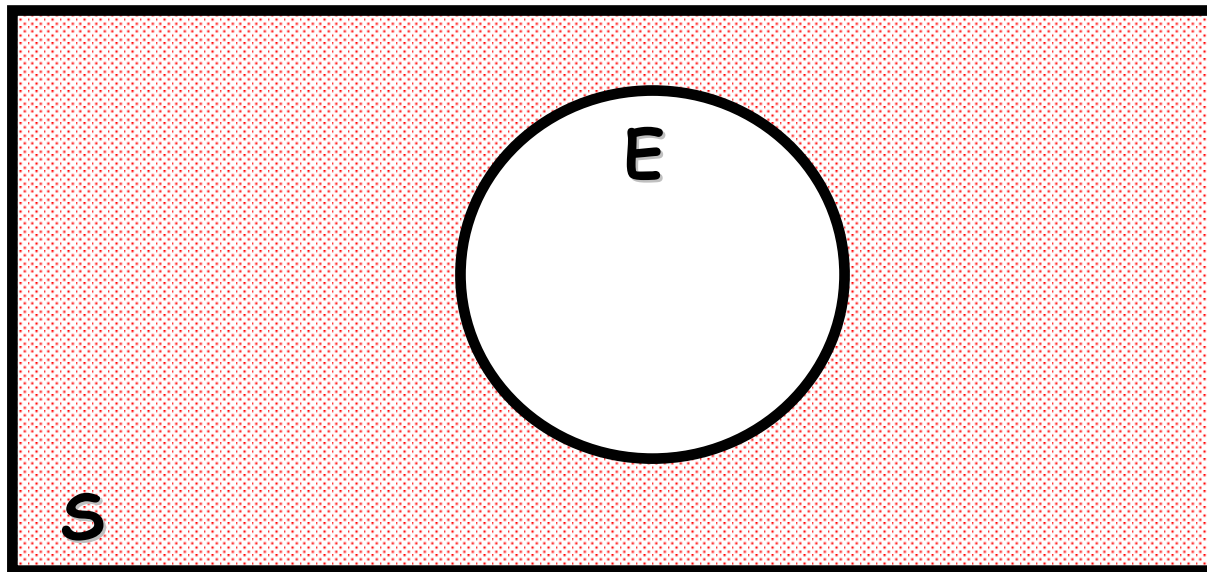
Venn Diagram for two Disjoint Events

- The two disjoint events, E and F , do not overlap.



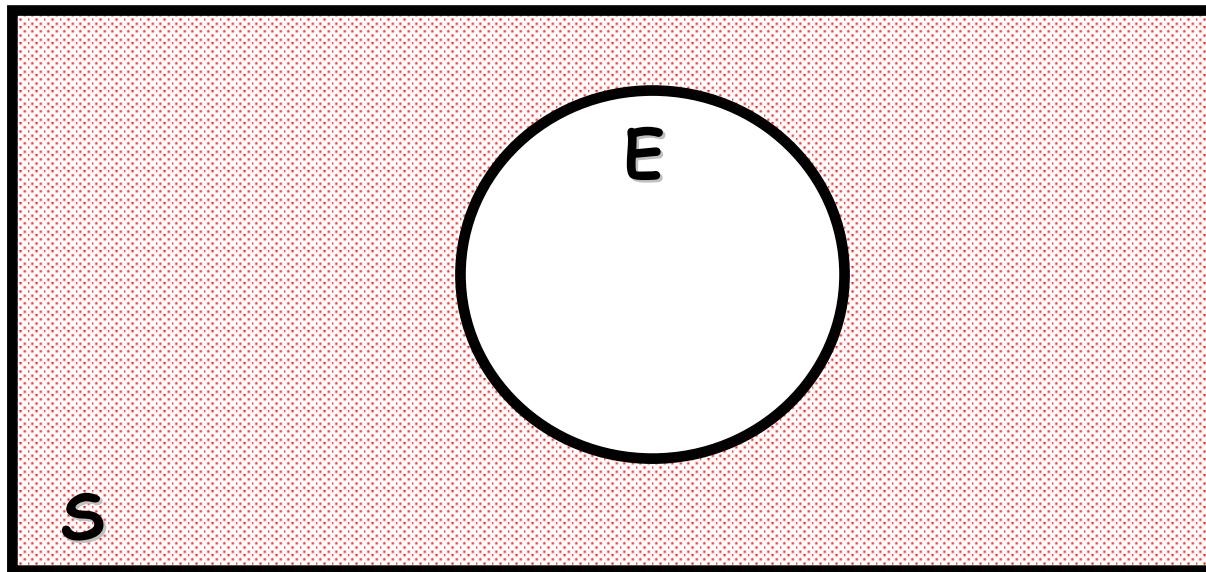
Complement of an Event

- The complement of event E , E' , and the event E are disjoint.



Complement of an Event

- The union of the event E and its complement E' is the sample space, S .
- $E \cup E' = S$

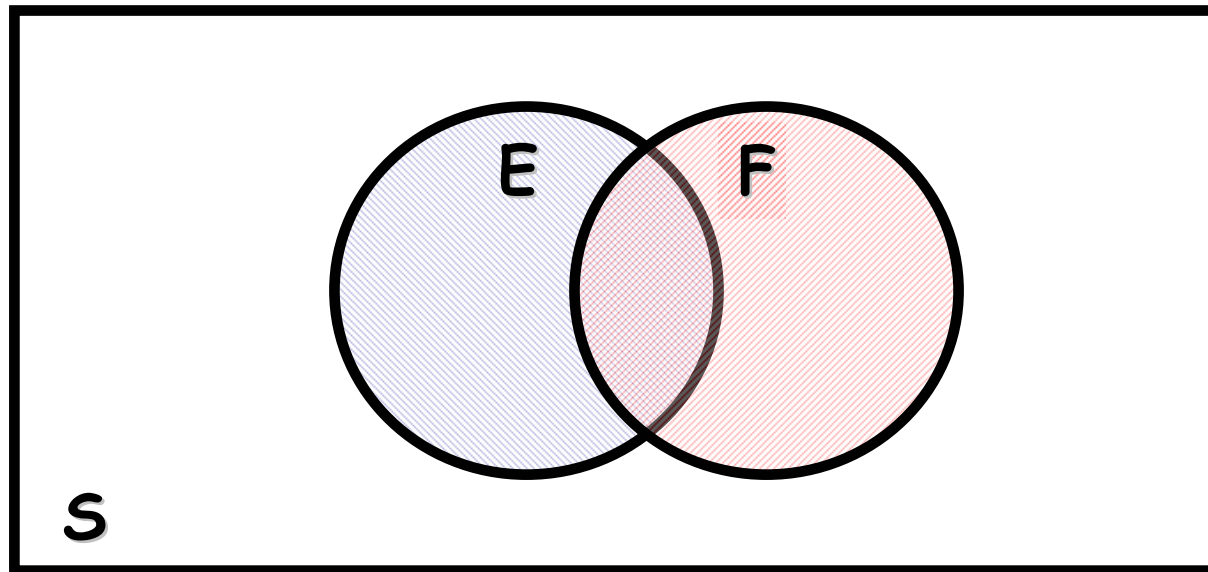


Non-Disjoint Events

- Two events, E and F , are *not* disjoint or *not* mutually exclusive if they have at least one simple event in common.

Venn Diagram for two Events that are not Disjoint

- The two events, E and F , are *not* disjoint if they overlap.

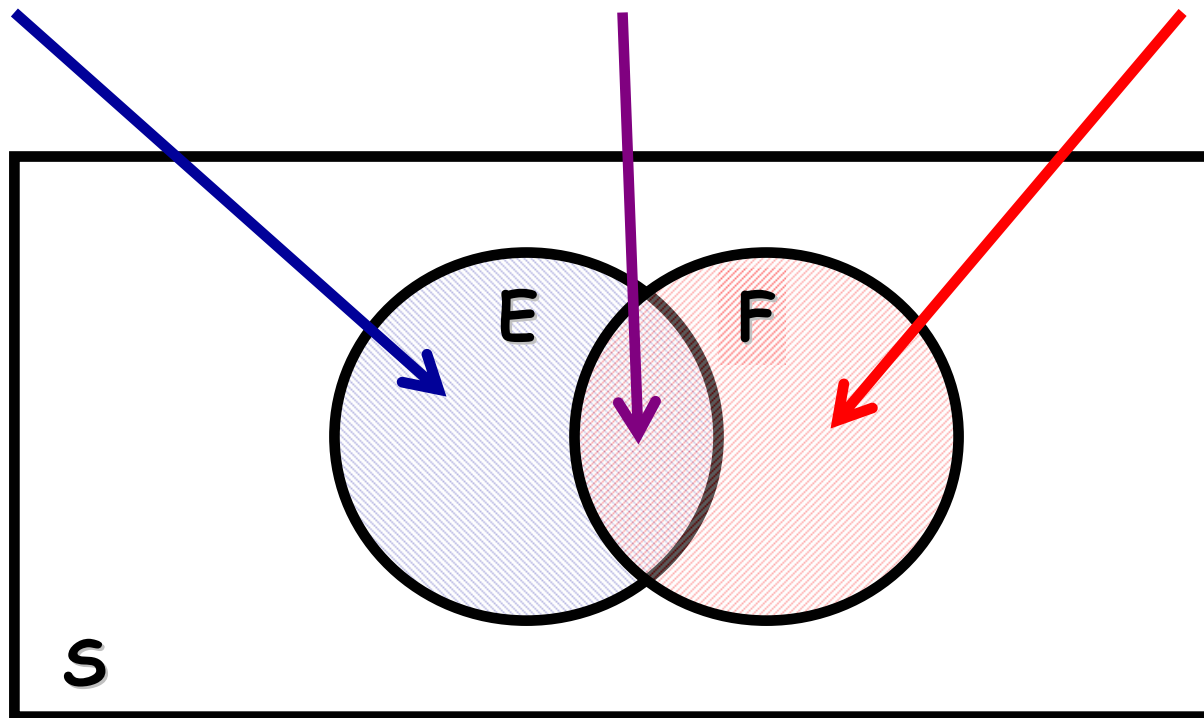


Venn Diagram for two Events that are not Disjoint

- In E and not in F

- In E and in F

- In F and not in E



$$E \cap F$$

- Given two events E and F , the event consisting of the simple events that the events E and F share or the simple events that events E and F have in common is denoted $E \cap F$.
- The event $E \cap F$ is the event that contains the simple events that are contained in *both* events E and F .

$$E \cap F$$

- $E \cap F$ denotes the intersection of events E and F
- The intersection of events E and F , $E \cap F$, is the set of simple events that are shared by events E and F
- The intersection of events E and F , $E \cap F$, is the set of simple events are in E and in F .

$$E \cap F$$

- $E \cap F$ denotes the *intersection* of events E and F
- The *intersection* of events E and F , $E \cap F$, is the set of simple events that are shared by events E and F
- The *intersection* of events E and F , $E \cap F$, is the set of simple events are in E *and* in F .
- KEY WORD: and

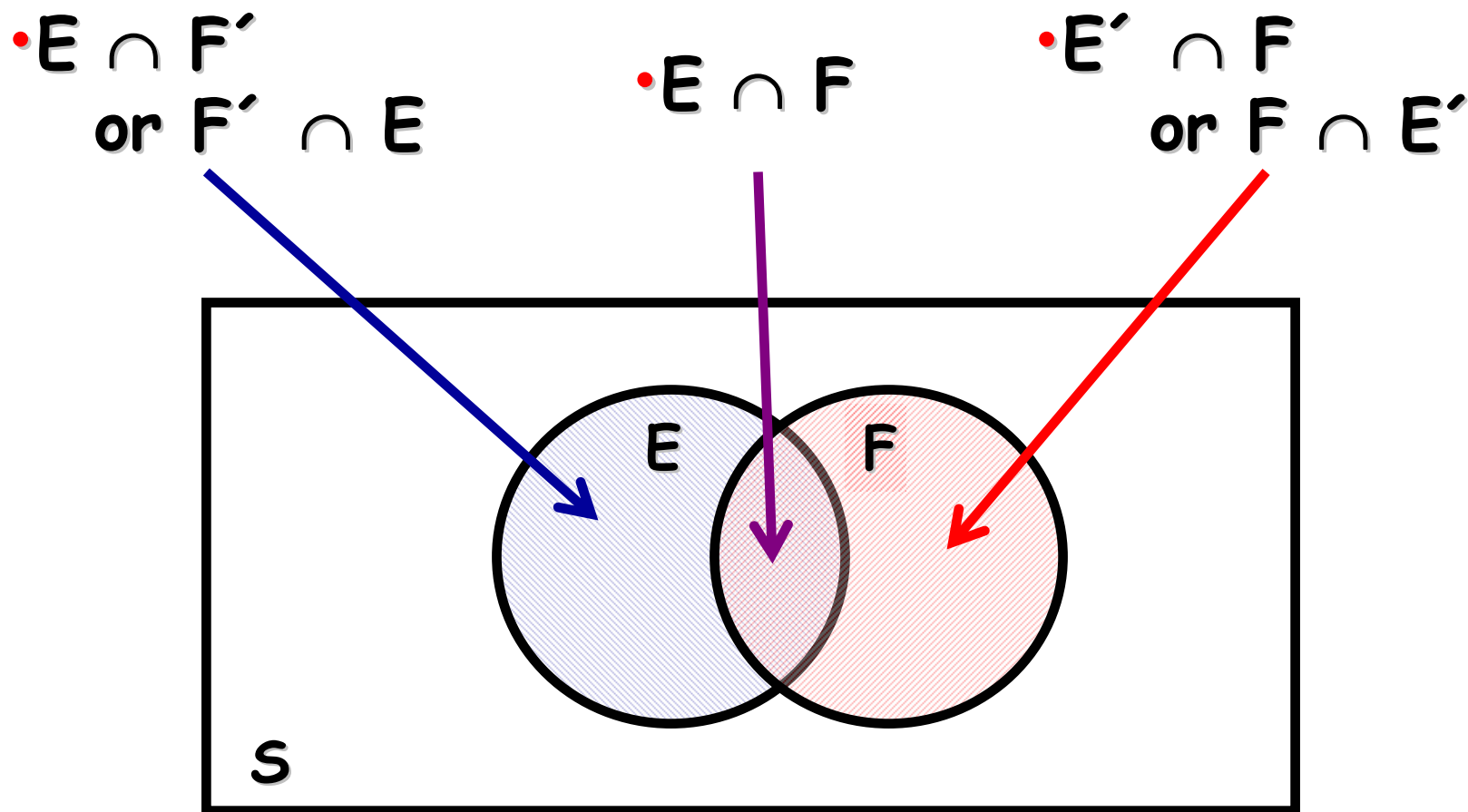
$$E \cap F'$$

- Given two events E and F , the event consisting of the simple events that are in event E and are not in event F is denoted $E \cap F'$.
- The event $E \cap F'$ is the event that contains the simple events that are contained in *both* events E and F' .

$$E' \cap F$$

- Given two events E and F , the event consisting of the simple events that are not in event E and are in event F is denoted $E' \cap F$.
- The event $E' \cap F$ is the event that contains the simple events that are contained in *both* events E' and F .

Venn Diagram for two Events that are not Disjoint



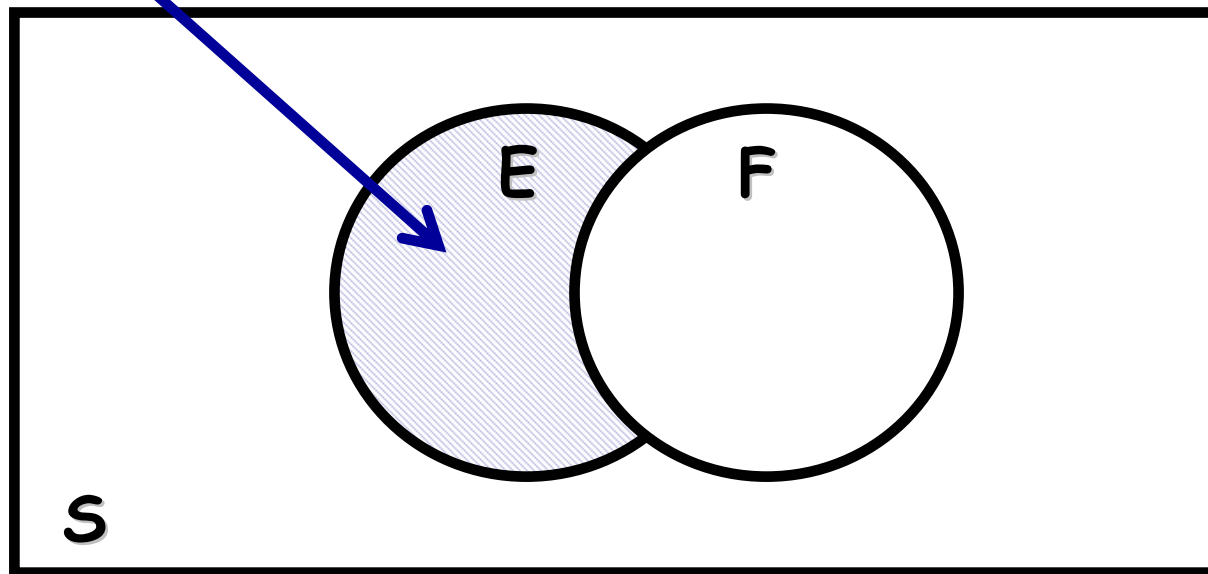
$$E \cup F$$

- Given two events E and F , the event consisting of the simple events that are contained in event E or contained in event F is denoted $E \cup F$.
- The event $E \cup F$ is the event that contains the simple events that are contained in event E or event F .

Each Part is contained in
 $E \cup F$ since each part is in
event E or in event F

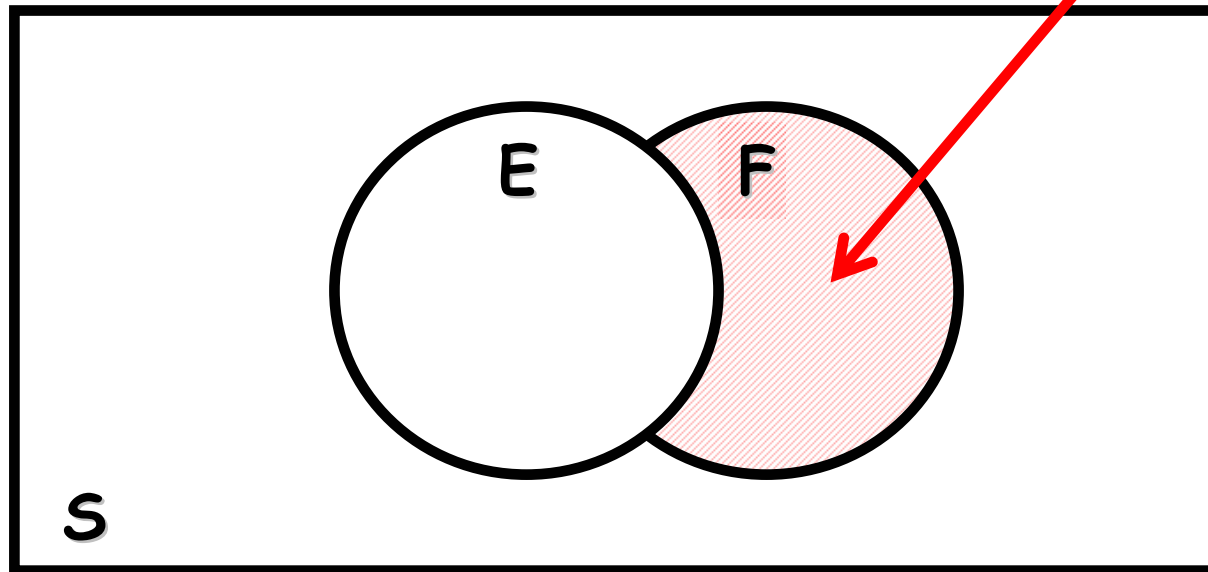
• $E \cap F'$
or $F' \cap E$

Note: This is the part of
 E that is *outside* of F



Each Part is contained in
 $E \cup F$ since each part is in
event E or in event F

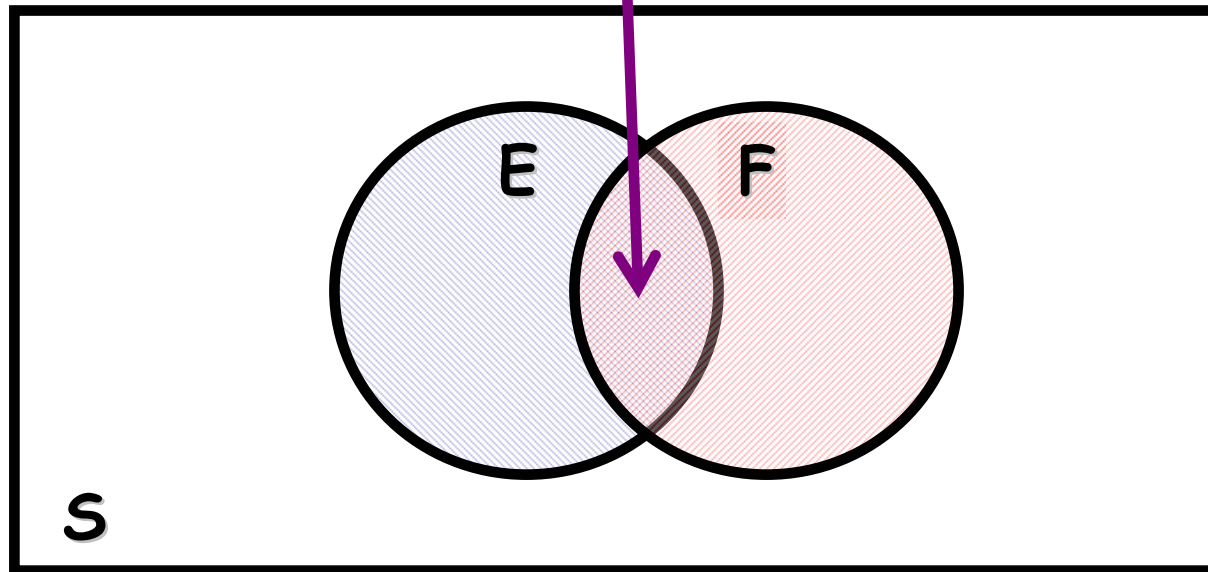
Note: This is the part of F • $E' \cap F$
that is *outside* E . or $F \cap E'$



Each Part is contained in
 $E \cup F$ since each part is in
event E or in event F

Note: This
part is in E .

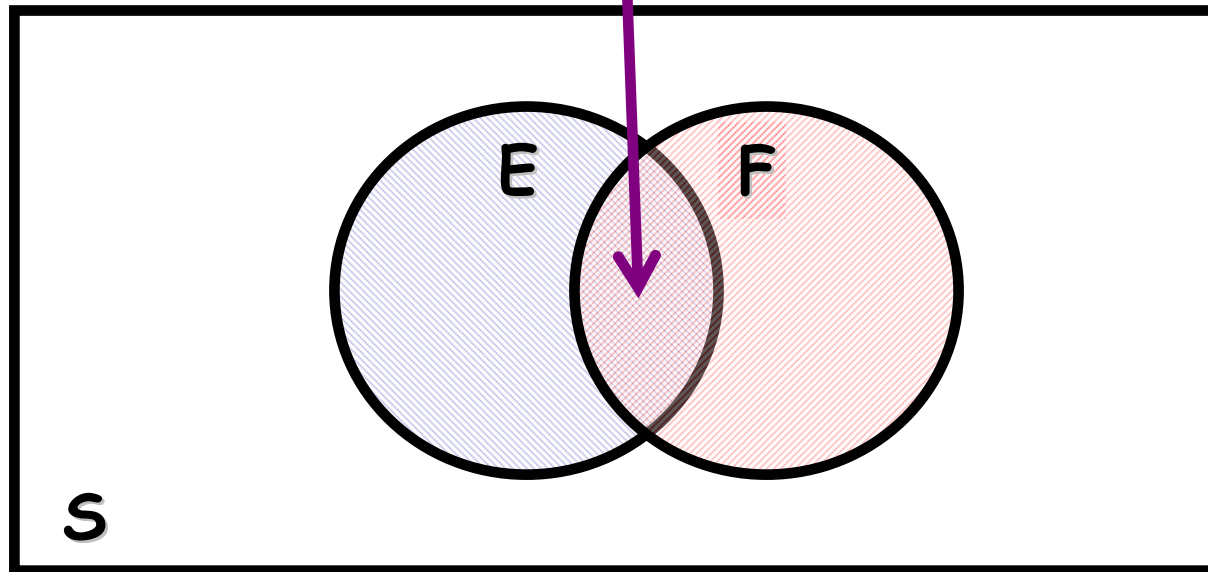
• $E \cap F$



Each Part is contained in $E \cup F$ since each part is in event E or in event F

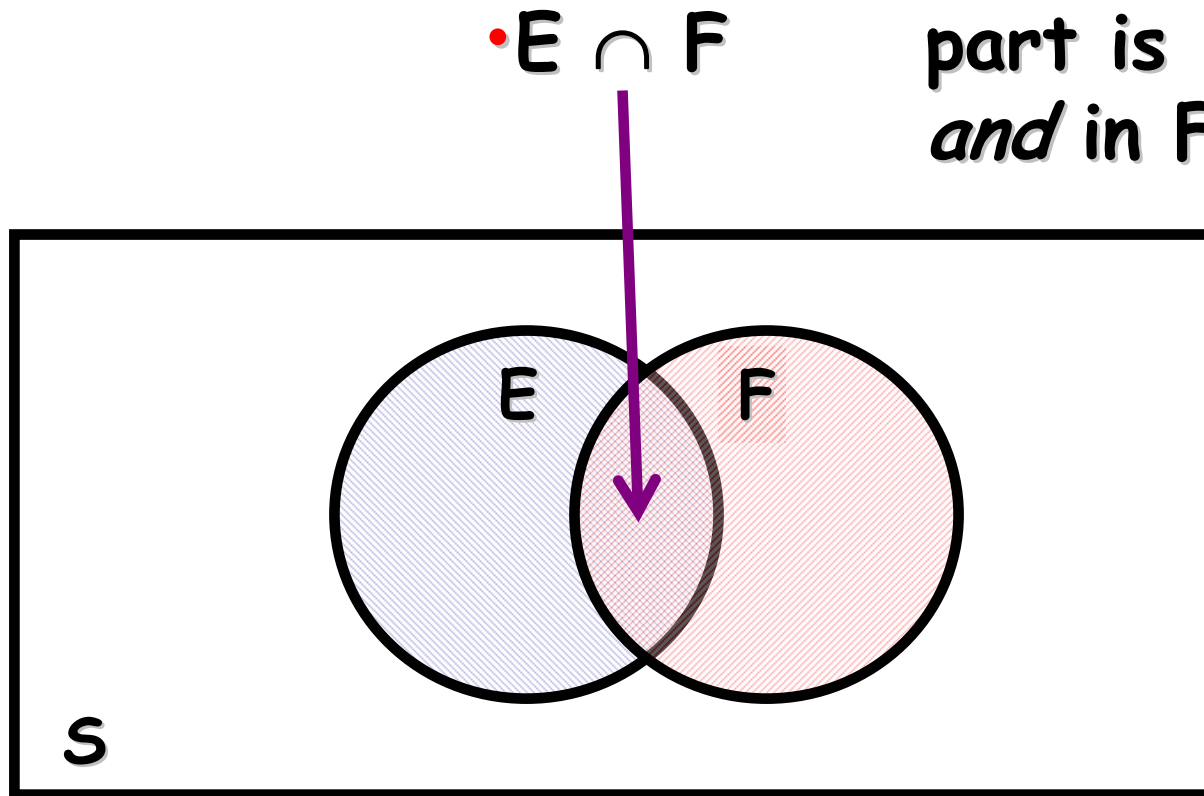
• $E \cap F$

Note: This part is in F .



Each Part is contained in $E \cup F$ since each part is in event E or in event F

Note: This part is in E and in F .

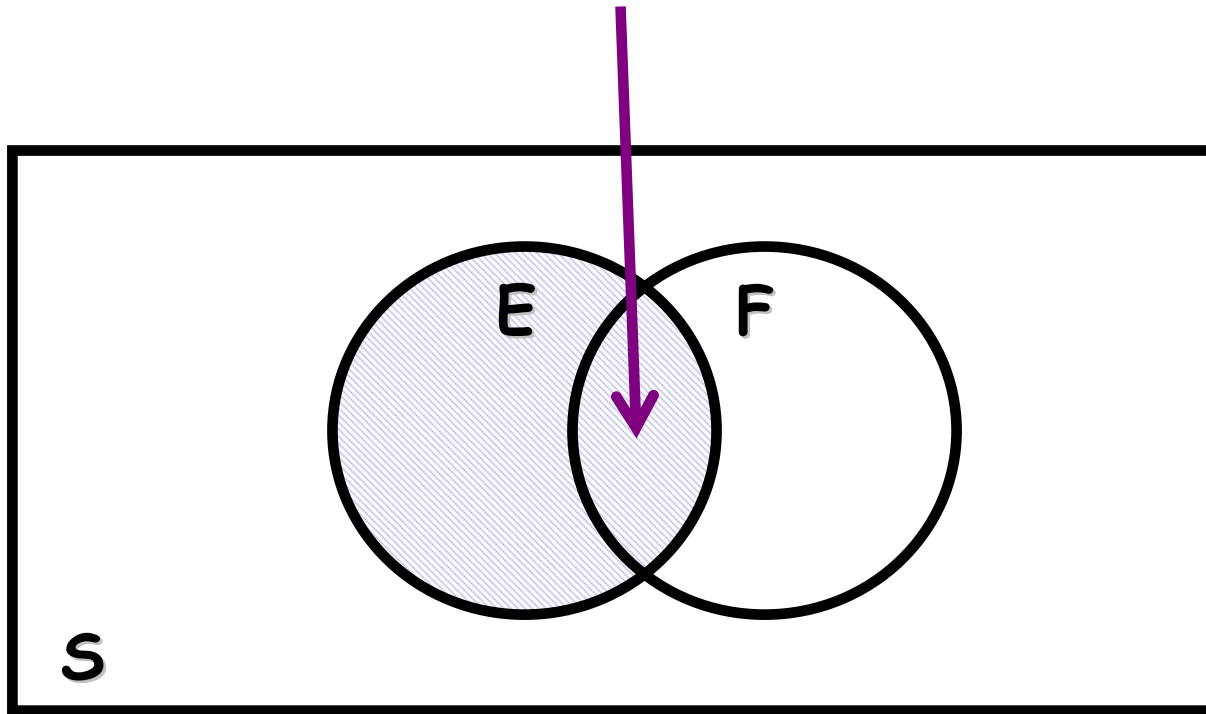


$$E \cup F$$

- $E \cup F$ denotes the *union* of events E and F .
- The *union* of events E and F , $E \cup F$, is the set of simple events that are in event E or in event F .
- The *union* of events E and F , $E \cup F$, is the set of simple events are event E , event F , or both event E and event F .
- KEY WORD: or

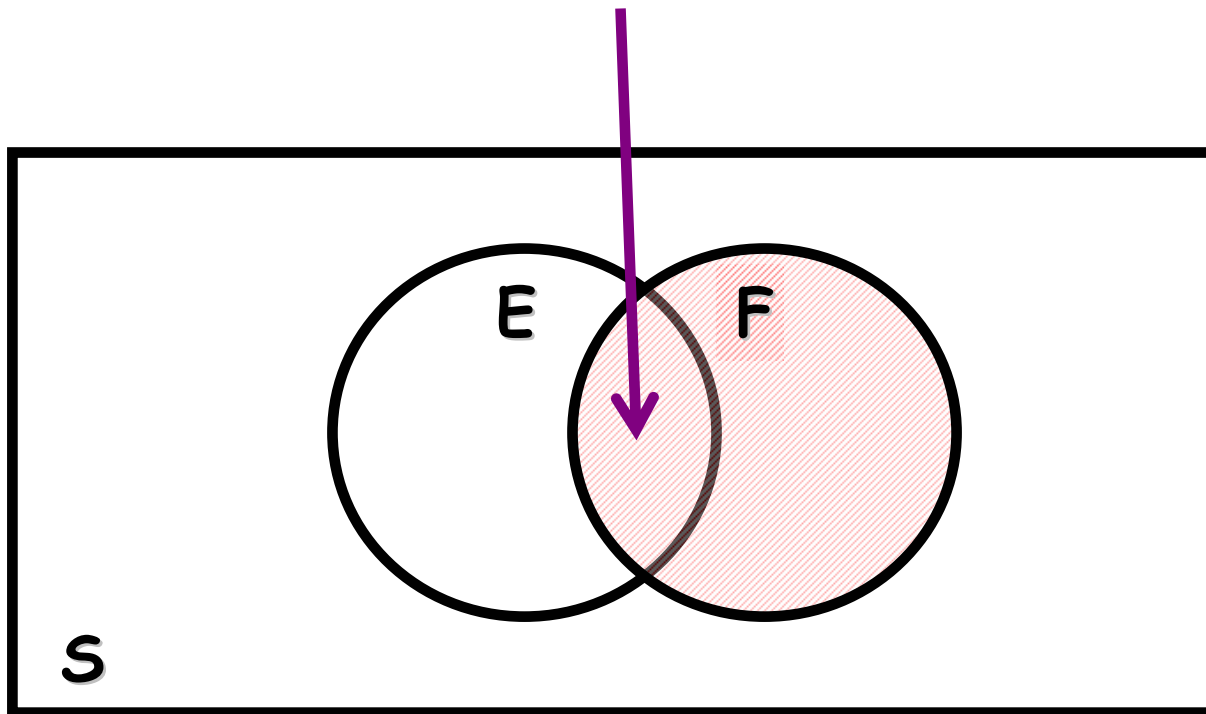
$E \cap F$ is contained in E

• $E \cap F$



$E \cap F$ is contained in F

• $E \cap F$

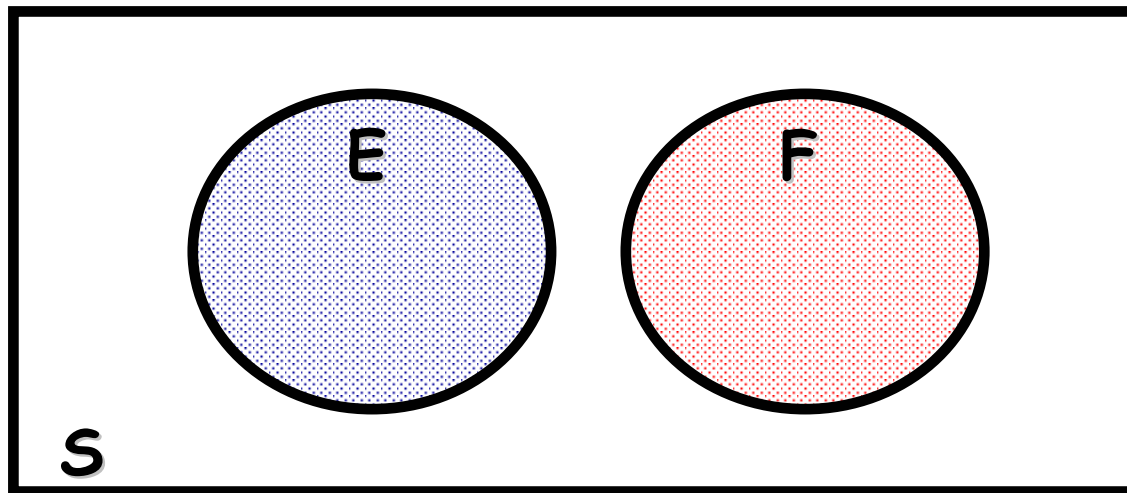


Probability of $E \cap F$

- There is no special rule for determining the probability of $E \cap F$
- To determine the $P(E \cap F)$, you must
 - Know the sample space
 - ◉ Know the number of simple events in the sample space
 - Know the event $E \cap F$
 - ◉ Know the number of simple events in the event $E \cap F$

Probability of $E \cup F$

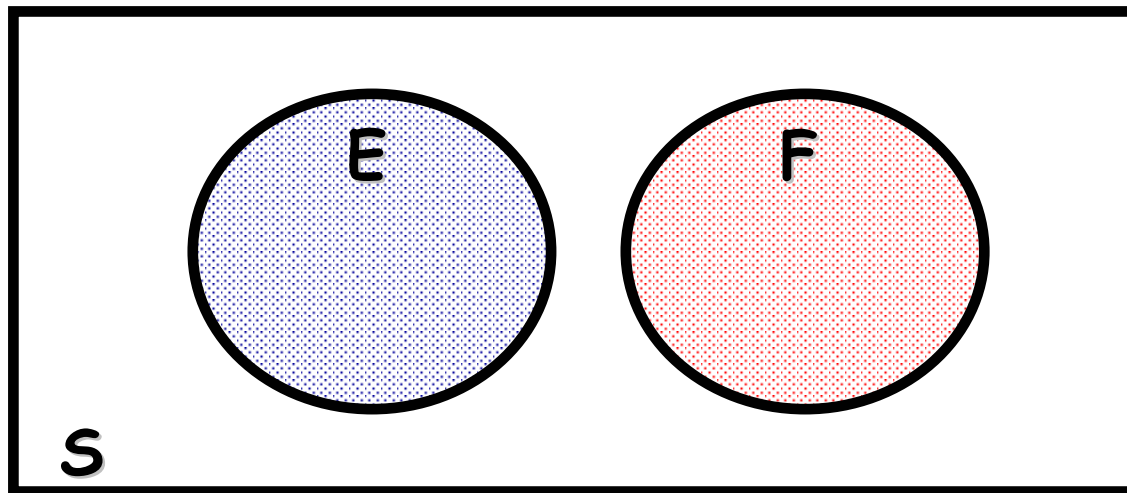
- Suppose events E and F are disjoint



- The probability that event $E \cup F$ occurs is the sum of the probabilities of the individual events E and F

Probability of $E \cup F$

- Suppose events E and F are disjoint



- $P(E \cup F) = P(E) + P(F)$

Example

- A fair die is rolled. What is the probability that the die displays a one or a five?

Example

- A fair die is rolled. What is the probability that the die displays a one or a five?
- Since the events *the die displays a one* and *the die displays a five* are disjoint,

$$\begin{aligned} P(\text{one or five}) &= P(\text{one}) + P(\text{five}) \\ &= 1/6 + 1/6 \\ &= 2/6 \\ &= 1/3 \end{aligned}$$

Example

- A card is drawn from a fair poker deck. What is the probability that the card is an Ace or a ten?

Example

- A card is drawn from a fair poker deck. What is the probability that the card is an Ace or a ten?
- Since the events *the card is an Ace* and *the card is a ten* are disjoint,
$$\begin{aligned} P(\text{Ace or ten}) &= P(\text{Ace}) + P(\text{ten}) \\ &= 4/52 + 4/52 \\ &= 8/52 \\ &= 2/13 \end{aligned}$$

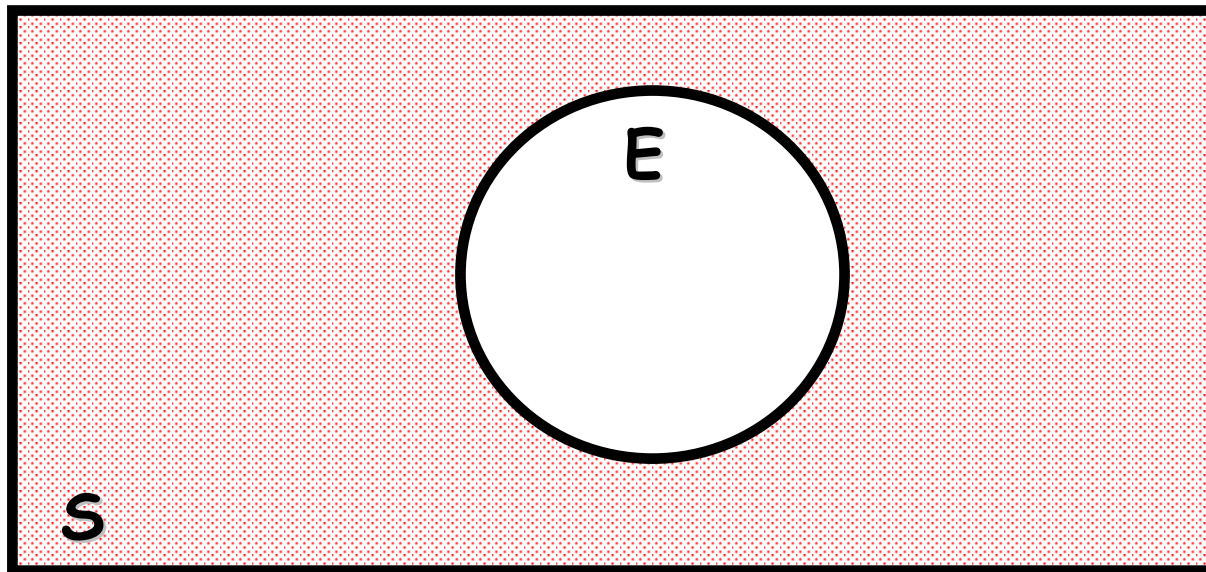
Example

- A card is drawn from a fair poker deck. What is the probability that the card is an Ace or a ten?
- Since the events *the card is an Ace* and *the card is a ten* are disjoint,
$$\begin{aligned} P(\text{Ace or ten}) &= P(\text{Ace}) + P(\text{ten}) \\ &= 1/13 + 1/13^* \\ &= 2/13 \end{aligned}$$

*The fractions are simplified first.

Probability of the Complement of an Event

- $P(E) + P(E') = P(S)$
- $P(E) + P(E') = 1$
- $P(E') = 1 - P(E)$



Example

- A fair die is rolled. What is the probability that the die does not display a five?

Example

- A fair die is rolled. What is the probability that the die does not display a five?
- $P(\text{not five}) = 1 - P(\text{five})$
 $= 1 - 1/6$
 $= 5/6$

Example

- A card is drawn from a fair poker deck. What is the probability that the card is not a face card?
- $P(\text{not face card}) = 1 - P(\text{face card})$
 $= 1 - 12/52$
 $= 40/52$
 $= 10/13$

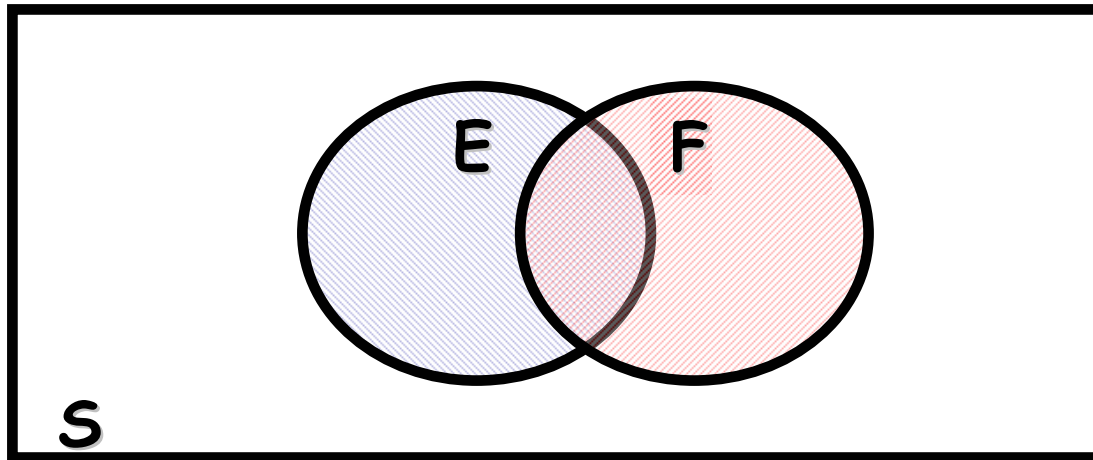
Example

- A card is drawn from a fair poker deck. What is the probability that the card is not a face card?
- $P(\text{not face card}) = 1 - P(\text{face card})$
 $= 1 - 3/13^*$
 $= 10/13$

*The fraction is simplified first.

Probability of $E \cup F$

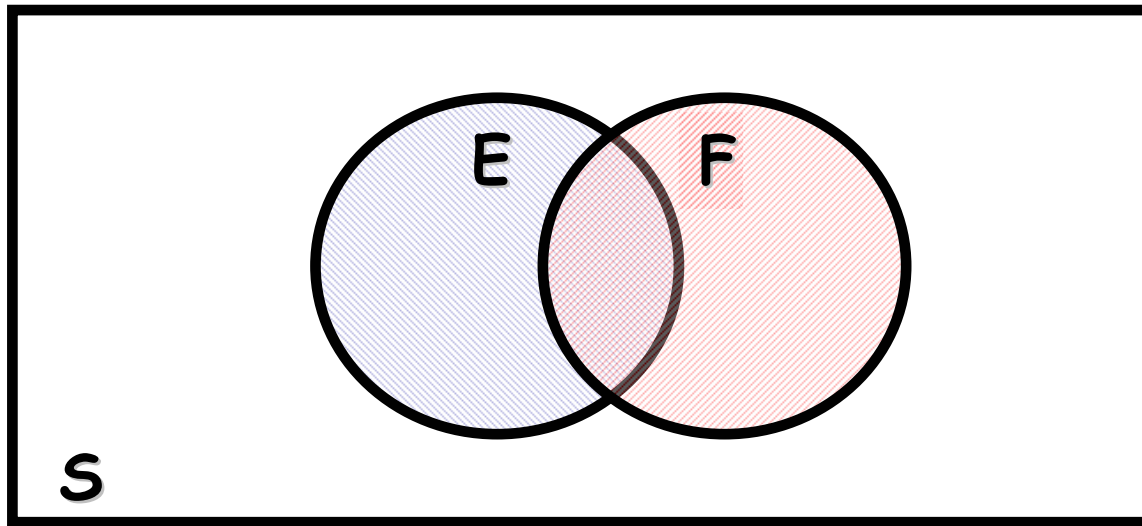
- Suppose events E and F are not disjoint



- The probability that event $E \cup F$ occurs is the sum of the probabilities of events E and F from which the probability of the intersection of event E and F , $E \cap F$, is *subtracted*

Probability of $E \cup F$

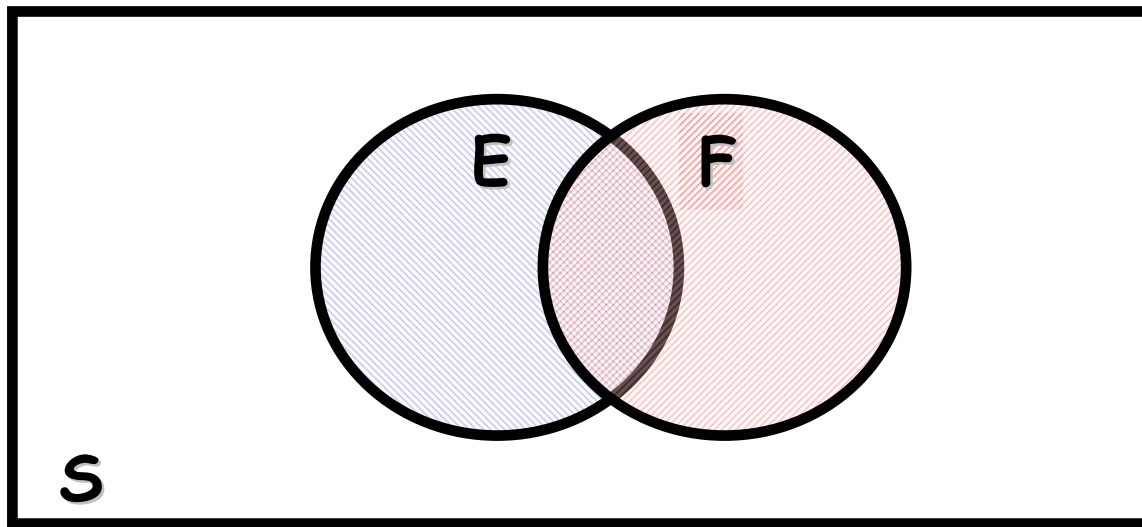
- Suppose events E and F are not disjoint



- $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

Probability of $E \cup F$ for non-disjoint events E and F

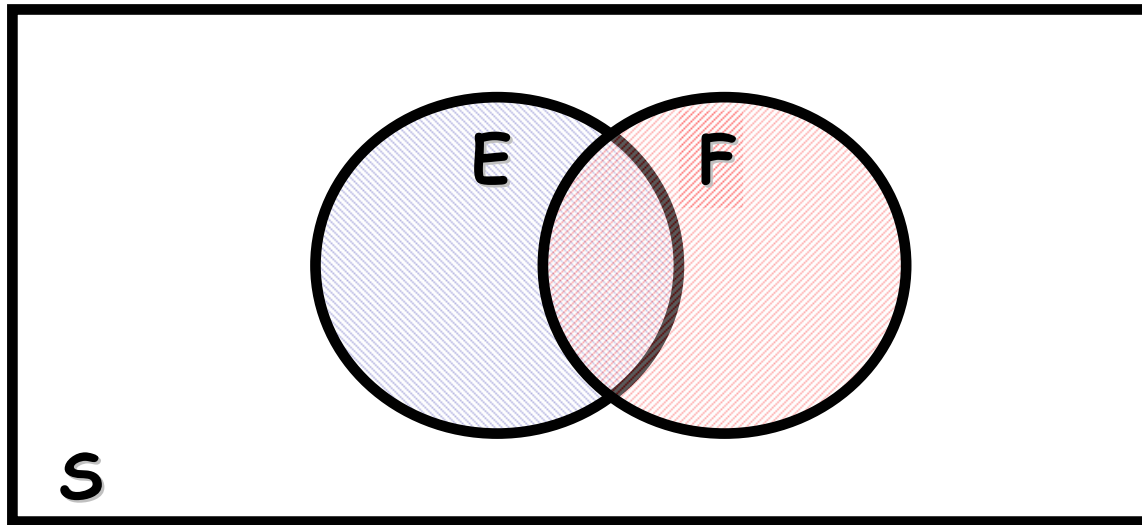
- $P(E \cup F) = P(E) + P(F) - P(E \cap F)$



- Why subtract $P(E \cap F)$?

Probability of $E \cup F$ for non-disjoint events E and F

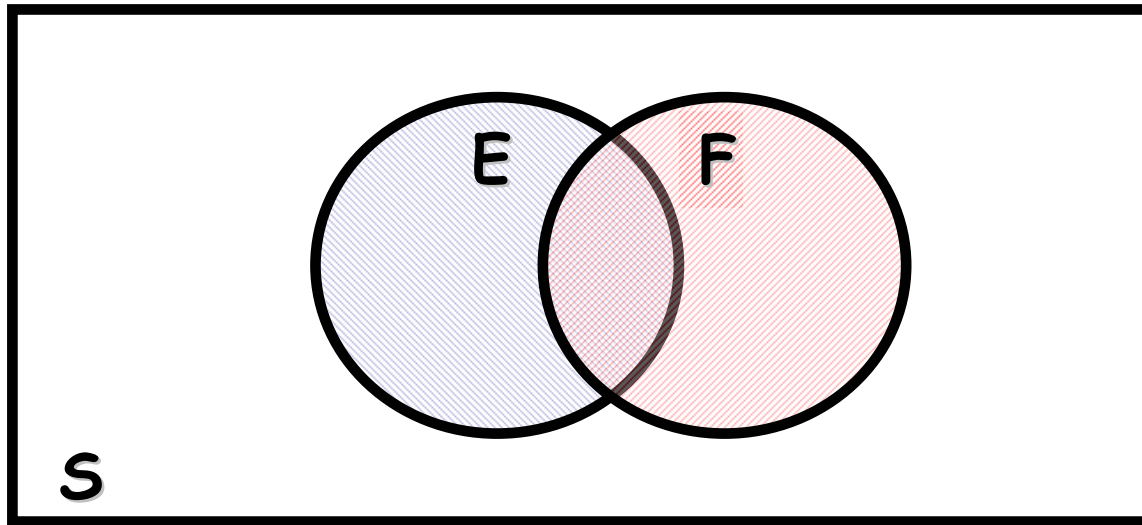
- Why subtract $P(E \cap F)$?



We subtract $P(E \cap F)$ since $E \cap F$ is contained in event E and $E \cap F$ is contained in event F .

Probability of $E \cup F$ for non-disjoint events E and F

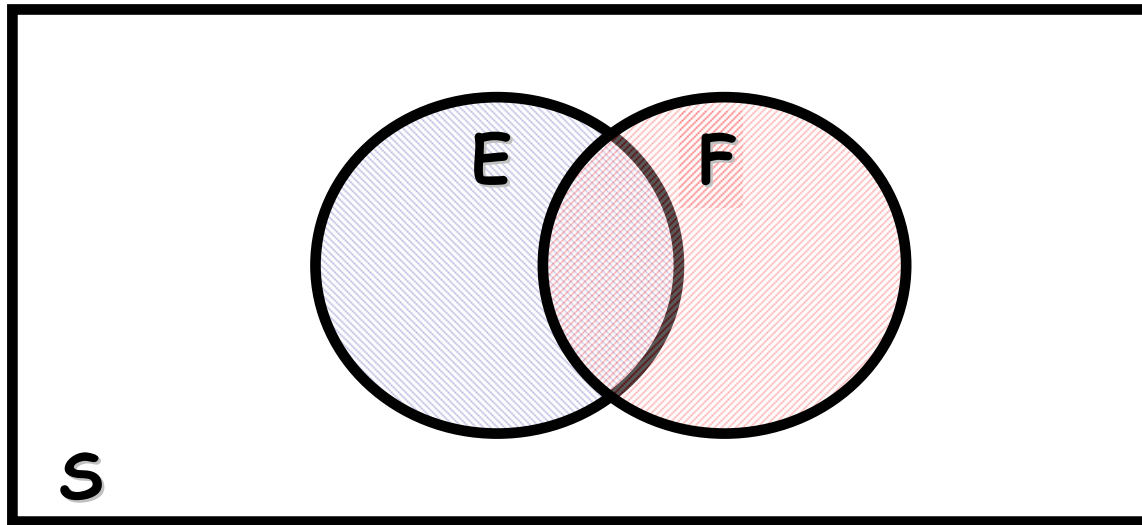
- Why subtract $P(E \cap F)$?



If we do not subtract $P(E \cap F)$ then $P(E \cup F)$ would count each of the simple events in $E \cap F$ *twice*.

Probability of $E \cup F$ for non-disjoint events E and F

- $P(E \cup F) = P(E) + P(F) - P(E \cap F)$



- This is also called the Addition Rule.

Example

- A card is drawn from a fair poker deck. What is the probability that the card is a diamond or a face card?
- $$\begin{aligned} P(\spadesuit \text{ or face card}) &= P(\spadesuit) + P(\text{face card}) \\ &\quad - P(\spadesuit \text{ face card}) \\ &= 13/52 + 12/52 \\ &\quad - 3/52 \\ &= 22/52 \\ &= 11/26 \end{aligned}$$

Example

- A card is drawn from a fair poker deck. What is the probability that the card is a diamond or a face card?
- $$\begin{aligned} P(\spadesuit \text{ or face card}) &= P(\spadesuit) + P(\text{face card}) \\ &\quad - P(\spadesuit \text{ face card}) \\ &= 1/4 + 3/13 \\ &\quad - 3/52^* \\ &= 22/52 \\ &= 11/26 \end{aligned}$$

*The fractions simplified first.