Proof Writing and Portfolios in a Bridge Course

Penny Dunham

Muhlenberg College

pdunham@muhlenberg.edu

Proof portfolios

- This talk will feature information about:
  - The course
  - Goals for portfolios in MTH 210
  - Initial instructions
  - Monitoring progress
  - Final instructions
  - Assessment process
  - Student comments/reactions

The setting

- Muhlenberg College
  - Four-year, liberal arts college
  - ~2200 students
  - undergrad only
- Math major
  - 10 math courses + 2 cognates
  - 10-22 math majors graduate per year
  - 5-10 minors/yr

The course

- MTH 210, Transition to Abstract Mathematics
  - gateway for almost all 300-level math courses
  - Two 75-minute meetings/week
- Text:
  - Smith, Eggen, & St. Andre: “A Transition to Advanced Mathematics”
- Audience
  - Mostly sophomore majors, some 1st & 3rd years; some minors
- Writing intensive: W-course in the major
  - Focus: how to write proofs
  - Critical/analytical writing
  - Revision/rewriting based on feedback
  - Writing for an audience

MTH 210 requirements

- Weekly assignments (25%)
  - 6-8 proofs, some computational problems
  - “proofs to grade” (text feature)
  - Required “rewrites” on proofs
- Class presentations w/peer review (10%)
- Quizzes (15%)
- Two short papers (10%)
  - Division properties
  - Mathematical induction
- Portfolio (40%)
  - Draft (10%)
  - Final with self-evaluative essay (30%)

Goals

- Through portfolios, students can:
  - experience writing as a process by
    - revising proofs
    - polishing their presentation
  - become aware of & document improvement
  - be reflective about good proof-writing while selecting best/representative work
  - ultimately, produce well-written and effective arguments

Initial instructions in syllabus

- At least twelve -- but no more than fifteen -- proofs from the graded assignments
- Proofs well-written in condensed prose style using complete sentences
- Concise but accurate exposition --avoid unnecessary steps; yet, include longer proofs as well as a few short ones.
- Items representative of the entire course, reflecting what you have learned about writing mathematical proofs.

Initial Instructions in syllabus

- The portfolio should include:
  - a wide selection of the major proof techniques
  - a variety of lengths from simple to complex arguments
  - proofs from a cross-section of the term's assignments (but not necessarily from every assignment)
  - proofs that you have rewritten (and improved!)
  - proofs that you found particularly interesting or challenging.

Monitoring progress

- Weekly graded proofs (score out of 10)
- Required re-submissions (score < 8)
- Feedback on math and writing
- Drafts for short papers
- Draft portfolio due 2 weeks before final submission (graded)

Final Instructions: Format

- Include a table of contents identifying the type of proof and the assignment number.
- With each item, include a beginning paragraph describing the item and explaining why you picked it as “representative” along with a copy of the original graded proof.
- Arrange the items in a folder or binder in the following order for each entry: (a) item description & justification, (b) rewritten proof, (c) graded original or photocopy of the graded proof.
- Descriptions and final versions of each proof must be typed. Original graded versions may be hand written.
- End with your self-evaluation essay.

Final Instructions:Self-evaluation

- Write a brief (~2 pages) reflective assessment of your growth in the course.
- You should:
  - Address what you have learned about proof writing
  - point to aspects of your portfolio that support your claims
- You may discuss (if you wish):
  - changes in your concept of what constitutes a proof
  - the role of proof in mathematics
  - aspects of the course that had an effect on your writing
- Remember: The essay is your chance to provide evidence of your progress over the term by telling me what you learned and pointing to specific examples in the portfolio that support your claim.

Assessment process

- Portfolios graded on:
  - mathematical correctness of the proofs
  - quality of writing
  - overall strength of selections
    - variety (proof style, length, complexity)
    - topic coverage
    - evidence of growth
  - reflection in essay
  - use of evidence in essay

Assessment process

Comments for content:

- Statement of theorem
- Appropriate proof method
- Logical development of argument
- Mathematical correctness
- Notation & symbol use

Comments for writing

- Organization and unity
  - Paragraph/sentence structure
  - Intro/transitions/conclusion
- Clarity
- Diction & word choice
- Basic Writing Errors (BWEs)
  - Sentence structure, grammar , spelling, punctuation

Student Reactions

Comments in essays demonstrate:

- Evidence of growth/improvement
- Reflection on strengths/weaknesses
- Insight into aspects of good proof-writing
- Writing as a process
  - Value of revision
  - Creation of portfolio
- Pride in accomplishments

Student comments: Evidence

- “My first attempt at proof #6 from Set 2 is unnecessarily long and includes redundant definitions, but my first attempt at proof #11 from Set 7 is extremely concise and includes only the essentials… My first attempt at proof #8 from Set 7 contained numerous errors, and my second attempt was still not completely correct. Even though I had written numerous proofs by that point in the course, I still struggled with certain proofs.” – A.D.

Student comments: Evidence

- “By contrasting the first proof in my portfolio with the last few, the increased complexity is quite obvious…My portfolio demonstrates my progression from elementary proofs concerning even/odd numbers and inequalities to more complex proofs dealing with cardinality and equivalence relations…. I was able to reduce the wordiness of proof 6 and used a theorem to shorten proof 10, making both proofs more concise. This is a challenge in proof writing: achieving both eloquence and terseness.”– M.H.

Student comments: Reflection

- “Near the beginning of the course, I would frequently use too many words and unnecessary steps in my proofs because I was not certain about the extent to which I had to justify my arguments. It was not immediately clear to me what constituted a valid proof…. However, as the semester went on, I learned to respect the intelligence of my reader and I realized that I was writing for an audience who had experience with math. Thus my proofs became more concise and efficient.” – A.M.

Student comments: Reflection

- “[T]he most improved part of my writing …is the use of my transitions within a paragraph…Prior to this semester, I had written in mostly in short, choppy sentences or with extremely long run-on sentences that ended up making no sense whatsoever. I learned through writing proofs that it is not smart to start off a sentence with a mathematical expression. Instead, it is more effective to use transitional introductions like ‘also,’ ‘furthermore,’ and ‘therefore.’ I then noticed while editing a paper I had written for my education class that I was using many of these phrases, and it made my writing easier to read and more cohesive.” – M.C.

Student comments: Reflection

- “This course has also improved my writing. Most importantly, it has drilled in me the importance of being specific, as vagueness is something I struggled with…This is a lesson I can carry on to my writing outside of math courses. As a writing tutor, I know the importance of being explicit and unambiguous, and this course helped emphasize that.” – L.S.

Student comments: Reflection

- When I was typing my proofs for the portfolio, I noticed small writing errors and changed them. While this might seem like a minor issue to some, it is an important one to me. When I can look back at old proofs and see writing issues right away, I realize how much better a writer I am now. I am more detail oriented now, and this not only helps writing, but it helps math, because math is all about details. – M.S.

Student comments: Insights

- “One of the most important attributes of a good proof, for instance, is its clarity and succinctness… The important thing is that the proof be precise and explain each step only with as many words as are necessary for the reader to proceed to the next part of the argument. A proof that is too confusing or verbose has failed to accomplish its task, even if it is based on completely sound mathematics.” – B.K.

Student comments: Insights

- “ Clarity is also an important issue when writing proofs. I’ve learned to write much more clearly when it comes to explaining the validity of a conjecture. I’ve learned when it is okay to be terse and when more is necessary…. Now I tend to be much more careful and detail oriented. Minor details are often the key to a successful proof.” – J.M.

Student comments: Process

- “I had the most difficulty with proofs by contradiction… For a while I didn’t understand what I needed to assume in order to reach a contradiction. Even in my rewrite, where I fixed the math errors, my prose form of writing is not something I would be proud of currently. When it came time to write the final draft for the portfolio, I edited this proof even further.” – M.R.

Student comments: Process

- “I have never been more frustrated and discouraged with a math class than I was with Transitions…It truly amazes me that I made it through this class alive. .. Yet, as I sit here trying to look back and assess my progress, I can honestly say that I have probably learned more math in this class than any other. ... I really like the idea of a portfolio because it gave me a second chance to revisit every kind of proof now that I feel I have a better grasp on the material. I really do love math, and I feel this course was a wonderful exploration into the beauty of it all.” –K.M.

Student comments: Pride

- “I remember working on some proofs for several hours… Even if I tried my hardest and then worked through the problem, I still did not get the right answer. Although this aspect was extremely frustrating, I still enjoyed how we were able to re-write the proofs….Overall, I particularly enjoyed this class and working on the portfolio. I am especially proud of all the proof styles that I am now able to use in future classes.” – A.T.

The Bottom Line

- Portfolios achieved goals of course:
  - Showed writing is a process
  - Developed proof-writing skills
- Students enjoyed creating portfolios:
  - Saw clear evidence of personal growth
  - Took great pride in their work
- Portfolios are effective tools to assess student learning in a bridge course.

Thank you!

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pdunham@muhlenberg.edu